

Title: Modeling Population Growth -- Growing by Leaps and Bounds!

Brief Overview:

Students will use population data to analyze linear growth. They will use their graphing calculators to construct scatter plots, perform linear regressions, and answer questions based on and about their model.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Algebra

Students will interpret representations of a function, and investigate rate of change and intercepts. They will use symbolic algebra to represent mathematical relationships. Students will be able to draw reasonable conclusions about the situation being modeled.

Geometry

Students will use Cartesian coordinates to represent the data and the model.

Data Analysis and Probability

Students will understand scatter plots, and use them to display data. They will determine regression equations that model the data using technological tools.

- **Process Standards**

Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation

These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will use authentic data to analyze and predict population growth. They must make appropriate conclusions, and use correct mathematical language to justify and explain their conclusions. Students will represent their data in tabular, graphical, and functional forms.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra

- **1.1.1**

Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**

Students will represent patterns and functional relationships in a table, as a graph, and by mathematical expression.

- **1.2.1**

Students will determine the equation for a line, solve linear equations, and describe the solutions using numbers, symbols and graphs.

- **1.2.4**

Students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Data Analysis and Probability

- **3.1.1**

Students will conduct an investigation that uses statistical methods to analyze data and communicate results.

- **3.2.1**

Students will make informed decisions and predictions based upon the results of data from research.

- **3.2.2**

Students will make predictions by finding and using a line of best fit.

- **3.2.3**

Students will communicate the use and misuse of statistics.

Grade/Level:

Grades 7-12; Algebra I

Duration/Length:

Three to five 45-minute lessons with pre-investigation assignment

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Constructing scatter plots
- Writing an equation of a line
- Graphing a line on a coordinate plane
- Understanding the units and meaning of slope and y-intercept
- Using a linear equation to determine domain and range values

Student Outcomes:

Students will:

- construct scatter plots using authentic data.
- determine and interpret regression equations using authentic data.
- use regression equations to model and predict population growth.
- use the list, regression, table, and plot functions of the graphing calculator.

Materials/Resources/Printed Materials:

- TI-83 or TI-83 Plus graphing calculator
- Data Analysis instructions for the graphing calculator
- Article “Decision on High School Site Pending” (included in this unit)
- Total enrollment figures for the instructor's school (Day 3 Investigation)
- Worksheets, Assessment

Development/Procedures:

Pre-Investigation Assignment

Students will read the article about the proposed location of the new Calvert County High School, and answer the questions. Students will also construct a scatter plot of the population of Calvert County over the last century.

Day 1

The following website lists census population data for all Maryland counties: <http://www.census.gov/population/cencounts/md190090.txt>. Students will investigate the population of Montgomery County from 1940 to 1990 using Worksheet 1. Students can collect the data directly from the website, or the instructor can give them the data. Students will use their calculators to construct a scatter plot, to find a regression line, and to make predictions.

Day 2

Students will look more closely at the population growth of Gaithersburg High School. They will use their calculators to construct a scatter plot, to find a regression line, to make predictions, and to draw conclusions regarding the feasibility of a new high school.

Day 3

Students will look more closely at the population growth of their own high school. They will use their calculators to construct a scatter plot, to find a regression line, to make predictions, and to draw conclusions regarding the need for a new high school.

Day 4

Students will be assessed on what they have learned with selected response, brief constructed response, and extended constructed response tasks.

Assessment:

This unit includes an assessment that consists of a selected response, a brief constructed response, and extended constructed response tasks. Tasks will assess how well students understand the vocabulary, skills, and reasoning processes of data analysis. Students will be asked to integrate data analysis, reasoning, and communication to support a conclusion that connects to a real-world application.

Extension/Follow Up:

- Students can research more data at www.census.gov for their own county.
- Students can research the data for their own school.
- Students can research articles for their own school or county.
- Students can revisit the data sets included in this unit later in the school year after having studied other types of functions, such as exponential, logistic, and piecewise.

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DECISION ON HIGH SCHOOL SITE PENDING; HOOK STRESSES NEED TO
RELIEVE CROWDING

Nancy Trejos
Washington Post Staff Writer

The Calvert County school board will soon decide which parcel of land to purchase for the county's fourth high school, and it is all but certain that site will be in the northern half of the county, despite an outcry from southern Calvert residents who say they are being unfairly deprived of a much-needed school.

"In my opinion, we need to get this issue behind us," Superintendent James R. Hook said during a luncheon with county commissioners and school board members on Tuesday.

Hook took pains during the session to convey to commissioners what he sees as an urgent need to build schools more quickly to relieve crowding. He once again told commissioners that even with a fourth high school in the works, the school system is not likely to be able to keep up with population growth without adding a fifth high school.

"When we open that high school in September 2004, it's going to be over capacity, and so will the other three high schools," Hook said.

Commissioners have been generally lukewarm to the idea of building a fifth high school, but on Tuesday, Commissioner Barbara A. Stinnett (D-At Large) said that option now seems inevitable.

"I think it's a reality that is going to have to be faced," she said.

After months of sitting on the sidelines as school board members debated where to build the fourth high school, county commissioners recently found the issue on their agenda, partly in response to residents complaining about the school board decision to place the school north of Prince Frederick.

Ultimately, the school board has the power to decide where to build the fourth high school. County commissioners must approve local funding, and at some point state officials have to give the green light to any location that is chosen.

Though at least one county commissioner has said he disagrees with the school board's decision to build in the north, on Tuesday Hook and school board members said it was time to put the disagreements behind them.

"We need to definitely get the fourth high school on the way," said Board of Education President MacArthur Jones, of Huntingtown.

The school board has narrowed its site choices to three parcels of land north of Prince Frederick and will soon make a decision on which to purchase, Jones said.

Earlier this month, the commissioners requested that county and school planners present them with studies on growth in the northern and southern parts of the county.

The statistical projections showed that most of the county's residents will be in the northern part of the county.

But some opponents of the school board decision, including Commissioner Patrick M. Buehler (D-St. Leonard), argued that growth would continue in the south, partly because less expensive homes there would attract younger families with school-age children.

However, school board member Robert L. Gray said the most influential determinant of the size of the school-age population will be the outcome of a redistricting plan.

"The key is going to be the redistricting," Gray said. "The location is important, but redistricting is more important."

Name _____
Date _____ Pd. _____

Pre-Investigation Assignment

Calvert County: Busting at the Seams!

You have read an article describing the types of considerations that counties face when placing a new school. Answer the following questions.

1. Will the fourth high school alleviate the school crowding in Calvert County? Cite the part of the article that answers this question.
2. What statistical evidence supports placing the new high school in the northern part of the county?
3. What is the argument that supports the proposition that growth in the southern part of the county will continue?

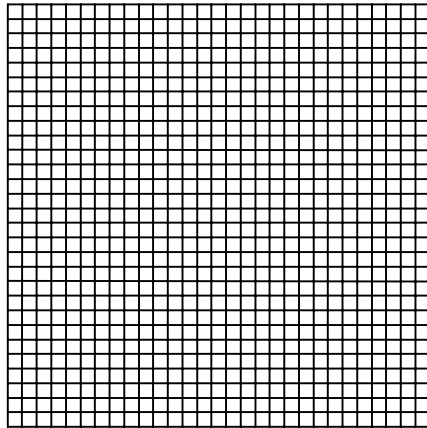
The population growth of Calvert County has increased rapidly over the last century. Let's take a closer look.

The United States conducts a census every ten years to learn more about its population and the needs of that population. Below is data collected from <http://www.census.gov/population/cencounts/md190090.txt>, listing the total population for Calvert County, Maryland, and the United States.

Population of Counties by Decennial Census: 1900 to 1990
Compiled and edited by Richard L. Forstall

	Calvert Co.	Maryland	U.S.
1900	10223	1188044	76212168
1910	10325	1295346	92228496
1920	9744	1449661	106021537
1930	9528	1631526	123202624
1940	10484	1821244	132164569
1950	12100	2343001	151325798
1960	15826	3100689	179323175
1970	20682	3922399	203211926
1980	34638	4216975	226545805
1990	51372	4781468	248709873

1. Which of the three columns shows the higher percentage of growth in population over the century? Defend your choice with mathematical reasoning.
2. Construct a scatter plot of the population growth in Calvert County. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below



3. Describe the population growth of Calvert County. Based on the trend, what do you predict the population will be in the 2000 census?

Name KEY
Date _____ Pd. _____

Pre-Investigation Assignment

Calvert County: Busting at the Seams!

You have read an article describing the types of considerations that counties face when placing a new school. Answer the following questions.

1. Will the fourth high school alleviate the school crowding in Calvert County? Cite the part of the article that answers this question.

No. "When we open that high school in September 2004, it's going to be over capacity, and so will the other three high schools."

2. What statistical evidence supports placing the new high school in the northern part of the county?

Statistical projections showed most of the county's residents will be in the northern part of the county.

3. What is the argument that supports the proposition that growth in the southern part of the county will continue?

Less expensive homes there would attract younger families with school-age children.

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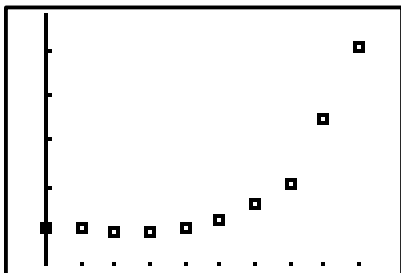
1. Which of the three columns shows the higher percentage of growth in population over the century? Defend your choice with mathematical reasoning.

$$\text{Calvert County} = (51372 - 10223) / 10223 = 403\% \text{ growth}$$

$$\text{Maryland} = (4781468 - 1188044) / 1188044 = 302\% \text{ growth}$$

$$\text{United States} = (248709873 - 76212168) / 76212168 = 226\% \text{ growth}$$

2. Construct a scatter plot of the population growth in Calvert County. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below



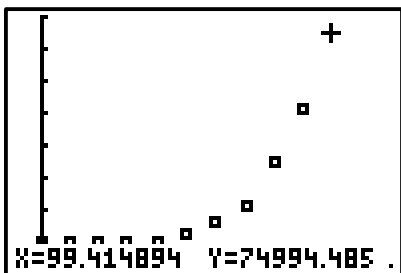
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WINDOW
Xmin=-9
Xmax=99
Xscl=10
Ymin=2414.52
Ymax=58485.48
Yscl=10000
Xres=1

```

3. Describe the population growth of Calvert County. Based on the trend, what do you predict the population will be in the 2000 census?

About 75,000, shown below.



[Expand the window to include $x=100$ for the year 2000, and higher populations on the y-axis. Press the arrow keys to move the cursor to a point that would approximate the next point in the trend.]

Data Analysis on the TI-83 or TI-83 PLUS

Entering Data:

1. Press STAT
2. Choose 1:Edit
3. To clear a list,
 - a) Press \uparrow to put the cursor on the list name
 - b) Press CLEAR
 - c) Press ENTER
4. Enter x's in L1 and y's in L2 (or other lists as appropriate)

Constructing a Scatter Plot:

1. Press 2^{ND} , Y=
2. Choose 1:Plot 1 (or other plot as appropriate)
3. Press ENTER to turn the plot ON
4. Press \downarrow , ENTER to choose a scatter plot
5. Press \downarrow , 2^{ND} , 1 to put L1 in the Xlist (or other list as appropriate)
6. Press \downarrow , 2^{ND} , 2 to put L2 in the Ylist (or other list as appropriate)
7. Press \downarrow , ENTER to choose the box mark
8. Press ZOOM
9. Choose 9:ZoomStat

Sketch your graph.

Press WINDOW to see the scale of the axes.

Finding a Best Fit Linear Model

1. Press STAT
2. Press \rightarrow to choose CALC
3. Choose 4:LinReg(ax+b)
4. To tell the calculator where your data is, and where to put results,
 - a) Press 2^{ND} , 1 to choose L1 for the x's (or other list as appropriate)
 - b) Press , (comma)
 - c) Press 2^{ND} , 2 to choose L2 for the y's (or other list as appropriate)
 - d) Press , (comma)
 - e) Press VARS
 - f) Press \rightarrow to choose Y-VARS
 - g) Choose 1:Function
 - h) Choose 1:Y1 (or other function as appropriate)

("LinReg(ax+b) L1, L2, Y1")

5. Press ENTER

Press GRAPH to see your model fit the data.

Worksheet 1

Montgomery County: Filling Up Fast!

Montgomery County also is experiencing a large growth rate. In 1998, two new high schools opened to help accommodate the growing population. The school system carefully watches the growth rate of the county in order to better predict where and when new schools will be needed. You have been hired by the county to find the growth rate.

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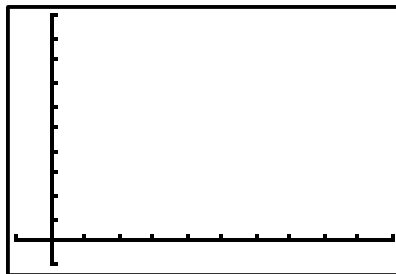
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1980	579053	4216975	226545805
1990	757027	4781468	248709873

(Use the graphing calculator instructions provided to help you answer the following questions.)

- Using your graphing calculator as a guide, sketch the relationship displaying the population of Montgomery County over time. Let $x=0$ represent 1940. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



- Using your graph, describe the population growth. What type of function best fits the trend? Why did you choose that function? Include units in your answer.
- Find the best-fit model (linear regression function) of the data. Record your results.
- Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.

- Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.
- Based on your model, prepare a brief report to Montgomery County officials stating your findings. Include predictions of the county population for the next two decades (2000, 2010). Include the rate of population growth. Be sure to include mathematical analysis to support your findings. Use another sheet of paper if necessary.
- Use your model to find the year when the county population will be approximately 800,000. Include mathematical analysis to support your findings.

Follow-Up Activity

Go to the website <http://www.census.gov/population/cencounts/md190090.txt>. Prepare a report about population growth for the county of your choice. Include all mathematical analysis and predictions. Communicate your findings in standard English. Be prepared to present your findings to the county council.

Worksheet 1

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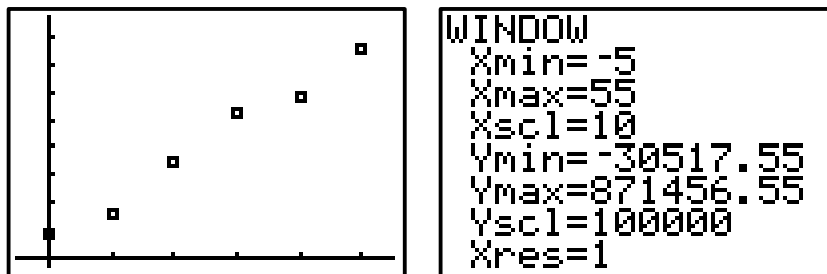
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(Use the graphing calculator instructions provided to help you answer the following questions.)

- Using your graphing calculator as a guide, sketch the relationship displaying the population of Montgomery County over time. Let $x=0$ represent 1940. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



- Using your graph, describe the population growth. What type of function best fits the trend? Why did you choose that function? Include units in your answer.

As time increases by decades, the population of Montgomery County increases. A linear model could fit the data given the steady rate of growth.

- Find the best-fit model (linear regression function) of the data. Record your results.

Let x = number of years since 1940

Let y = population of Montgomery County

$$y = 13689.748571429x + 65777.95238094$$

4. Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.

The positive slope indicates an increase of approximately 13,689.75 people per decade in Montgomery County.

5. Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.

The y-intercept is approximately 65,777.95, which represents the number of people in Montgomery County when $x = 0$, or 1940. This is the model's prediction, but the actual value according to the data is 83,912.

6. Based on your model, prepare a brief report to Montgomery County officials stating your findings. Include predictions of the county population for the next two decades (2000, 2010). Include the rate of population growth. Be sure to include mathematical analysis to support your findings. Use another sheet of paper if necessary.

Predictions for	2000	$x = 60$	$y = 887,163$
	2010	$x = 70$	$y = 1,024,060$

Rate of population growth = slope = 13,689.75 people per decade

Reports will vary.

7. Use your model to find the year when the county population will be approximately 800,000. Include mathematical analysis to support your findings.

$800000 = 13689.748571429x + 65777.95238094$
 $x = 53.6$ @ between 1993 and 1994

The county population will be approximately 800,000 just before the year 1994.

Follow-Up Activity

Go the website <http://www.census.gov/population/cencounts/md190090.txt>. Prepare a report about population growth for the county of your choice. Include all mathematical analysis and predictions. Communicate your findings in standard English. Be prepared to present your findings to the county council.

Worksheet 2

Gaithersburg High School: Student Surplus!

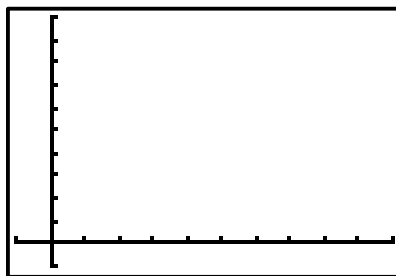
Montgomery County's population growth has been reflected in its schools' enrollments year after year. The growing county has recently opened two new high schools, and plans to open more. County officials watch carefully the locations with the most growth to help plan where new high schools should be built. Gaithersburg High School, one of the larger high schools in Montgomery County, has been serving its community for almost 100 years. Its population has grown steadily over the years.

Below is data collected from Montgomery County Public Schools administration for Gaithersburg High School's total enrollment since 1989:

	Gaithersburg HS
1989	1592
1990	1372
1991	1421
1992	1629
1993	1721
1994	1780
1995	1801
1996	1758
1997	1860
1998	1839
1999	1897

(Use the graphing calculator instructions provided to help you answer the following questions.)

- Using your graphing calculator as a guide, sketch the relationship displaying the population of Gaithersburg High School over time. Let $x=0$ represent 1989. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



- Using your graph, describe the population growth. Make particular note of where the growth increased, slowed, or decreased. Suggest reasons why this might have happened.

3. What type of function best fits the trend? Why did you choose that function?
4. Find the best-fit model (linear regression function) of the data. Record your results.
5. Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.
6. Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.
7. Based on your model, prepare a brief report to Montgomery County officials stating your findings. Include predictions of the school population for the next two years. Include the rate of population growth. Be sure to include mathematical analysis to support your findings. Use another sheet of paper if necessary.
8. Montgomery County has lost the data for 1988. Use your model to extrapolate and find the population for the missing year. Include mathematical analysis to support your findings.
9. Find the year when the population of Gaithersburg High School will be 2,050. Include mathematical analysis to support your findings.

Follow-Up Activity

Find the total enrollment for your own high school for as many years as specified by your instructor. You will use the data to complete a similar analysis in class.

Worksheet 2

Gaithersburg High School: Student Surplus!

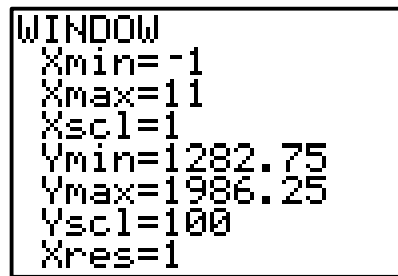
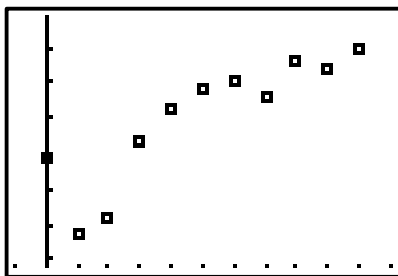
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(Use the graphing calculator instructions provided to help you answer the following questions.)

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- Using your graph, describe the population growth. Make particular note of where the growth increased, slowed, or decreased. Suggest reasons why this might have happened.

Since 1989, the population of Gaithersburg High School has increased. There were decreases in the population between 1989-90, 1995-96, and 1997-98. These decreases could have been caused by new schools opening in the area, depleting the Gaithersburg HS population.

3. What type of function best fits the trend? Why did you choose that function?
A linear model could fit the data given the steady rate of growth.
4. Find the best-fit model (linear regression function) of the data. Record your results.
Let x = number of years since 1989
Let y = population of Gaithersburg High School

$$y = 45.890909090909x + 1467.8181818182$$
5. Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.
The positive slope indicates an increase of approximately 45.89 people per year in Gaithersburg High School.
6. Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.
The y-intercept is approximately 1467.82, which represents the number of students in Gaithersburg High School when $x = 0$, or 1989. This is the model's prediction, but the actual value according to the data is 1592.
7. Based on your model, prepare a brief report to Montgomery County officials stating your findings. Include predictions of the school population for the next two years. Include the rate of population growth. Be sure to include mathematical analysis to support your findings. Use another sheet of paper if necessary.
Predictions for

2000	$x = 11$	$y = 1,973$
2001	$x = 12$	$y = 2,019$

Rate of population growth = slope = 45.89 students per year

Reports will vary.
8. Montgomery County has lost the data for 1988. Use your model to extrapolate and find the population for the missing year. Include mathematical analysis to support your findings.
1988 @ $x = -1$

$$y = 45.890909090909(-1) + 1467.8181818182$$

$$y = 1422$$

Gaithersburg High School had approximately 1,422 students in 1988.
9. Find the year when the population of Gaithersburg High School will be 2,050. Include mathematical analysis to support your findings.

$$2050 = 45.890909090909x + 1467.8181818182$$

$$x = 12.7 \text{ @ between 2001 and 2002}$$

The school population will be approximately 2,050 just before the year 2002.

Follow-Up Activity

Find the total enrollment for your own high school for as many years as specified by your instructor. You will use the data to complete a similar analysis in class.

Worksheet 3
Your High School: Path to the Future

Your school district has hired you based on your expertise in population growth. School officials are interested in your school's rate of growth. You must prepare a report that will recommend to the district whether there is a need for expansion.

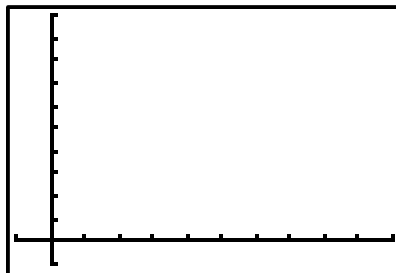
Construct a table below that shows the enrollment for your school over several years.

Year	Enrollment

•
•
•

(Use the graphing calculator instructions provided to help you answer the following questions.)

- Using your graphing calculator as a guide, sketch the relationship displaying the population of _____ High School over time. Let $x=0$ represent the first year. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



- Using your graph, describe the population growth. Make particular note of where the growth increased, slowed, or decreased. Suggest reasons why this might have happened.
- What type of function best fits the trend? Why did you choose that function?

4. Find the best-fit model (linear regression function) of the data. Record your results.
5. Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.
6. Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.
7. Based on your model, prepare a brief report to school officials stating your findings. Include predictions of the school population for the next two years. Include the rate of population growth. Be sure to include mathematical analysis to support your findings and recommendations. Use another sheet of paper if necessary.
8. Find the year when the population of _____ High School will be twice what it is now. Include mathematical analysis to support your findings.

Follow-Up Activity

Review the procedures and skills you have used. Be prepared to complete several assessment tasks.

Worksheet 3
Your High School: Path to the Future

Your school district has hired you based on your expertise in population growth. School officials are interested in your school's rate of growth. You must prepare a report that will recommend to the district whether there is a need for expansion.

Construct a table below that shows the enrollment for your school over several years.

Year	Enrollment

•
•
•

(Use the graphing calculator instructions provided to help you answer the following questions.)

- Using your graphing calculator as a guide, sketch the relationship displaying the population of _____ High School over time. Let $x=0$ represent the first year. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

- Using your graph, describe the population growth. Make particular note of where the growth increased, slowed, or decreased. Suggest reasons why this might have happened.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

- What type of function best fits the trend? Why did you choose that function?

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

4. Find the best-fit model (linear regression function) of the data. Record your results.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

5. Identify the slope of your model. What does it mean in the context of this problem, and is it meaningful? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

6. Identify the y-intercept of the model. What does it mean in the context of this problem, and is it meaningful? Include units in your answer.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

7. Based on your model, prepare a brief report to school officials stating your findings. Include predictions of the school population for the next two years. Include the rate of population growth. Be sure to include mathematical analysis to support your findings and recommendations. Use another sheet of paper if necessary.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

8. Find the year when the population of _____ High School will be twice what it is now. Include mathematical analysis to support your findings.

Answers will vary, but the procedures will be similar to Worksheet 2: Gaithersburg High School.

Follow-Up Activity

Review the procedures and skills you have used. Be prepared to complete several assessment tasks.

Assessment

Teacher's Guide

Introduction

This performance assessment is intended to be used after the unit “Modeling Population Growth” has been completed. The assessment consists of a combination of selected response items, a brief constructed response, and an extended constructed response.

Objectives Covered

After the unit has been given, students should be able to do the following:

- use the graphing calculator to construct scatter plots and perform linear regressions.
- use population data to analyze linear growth.
- answer questions based on and about a linear mathematical model.

Tools/Materials Needed for Assessment

- TI-83 or TI-83 Plus graphing calculator
- Instructions for using the graphing calculator

Administering the Assessment

Students should be allowed approximately 45 minutes to complete the assessment.

Name _____
Date _____ Pd. _____

Assessment
Modeling Population Growth

Part I

Place a circle around the letter of the correct answer for problems 1-3.

Dr. Mildred Trotter, a physical anthropologist, determined the relationship between a person's femur (thighbone) length and his/her height. The equation she developed to predict height in centimeters, H , based on femur length in centimeters, F , is

$$H = 2.38F + 61.41$$

Use this model to answer the following questions.

1. Anthropologists found the femur of a woman to be 41.5 cm. Determine her height.
a) 102.91 cm b) 105.29 cm c) 108.37 cm d) 160.18 cm
2. Mr. Day is 180.4 cm tall. What is the length of his femur?
a) 14.39 cm b) 50.00 cm c) 101.60 cm d) 116.61 cm
3. Jason's femur measures 40 cm. John's femur measures 42 cm. What is the difference in their predicted heights?
a) 0.90 cm b) 2.00 cm c) 4.76 cm d) 66.17 cm

Part II

The proportion of a fourteen year old's height to head length is 7 to 1. This relationship can be written as

$$H = 7L$$

where H is the height and L is the length of the head.

1. Complete the following table using this model.

Head Length	Height
5	35
8	
15	
17	
	161
	133

2. Using your graphing calculator as a guide, sketch the relationship displaying #1 above. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



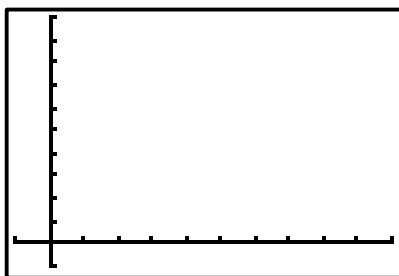
3. Describe the relationship between height and head length. Include units in your answer.

Part III

You have been hired by ImagineLand County to analyze its population growth. Below is the population over the last five years.

Year	Population (in 100,000's)
1995	6
1996	7
1997	8.5
1998	8.8
1999	12.5

1. Using your graphing calculator as a guide, sketch the relationship displaying the population. Let $x=0$ represent 1995. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below



2. Find the best-fit model (linear regression) of the data. Record your results.
3. Identify the slope of your model. What does it mean in the context of this problem? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.
4. What will you report as the population growth rate to the ImagineLand County Council?
5. Predict the population of ImagineLand County in the year 2005. Include mathematical analysis to support your findings.

Name KEY
Date _____ Pd. _____

Assessment
Modeling Population Growth

Part I

Place a circle around the letter of the correct answer for problems 1-3.

Dr. Mildred Trotter, a physical anthropologist, determined the relationship between a person's femur (thighbone) length and his/her height. The equation she developed to predict height in centimeters, H, based on femur length in centimeters, F, is

$$H = 2.38F + 61.41$$

Use this model to answer the following questions.

1. Anthropologists found the femur of a woman to be 41.5 cm. Determine her height.
a) 102.91 cm b) 105.29 cm c) 108.37 cm **d) 160.18 cm**
2. Mr. Day is 180.4 cm tall. What is the length of his femur?
a) 14.39 cm **b) 50.00 cm** c) 101.60 cm d) 116.61 cm
3. Jason's femur measures 40 cm. John's femur measures 42 cm. What is the difference in their predicted heights?
a) 0.90 cm b) 2.00 cm **c) 4.76 cm** d) 66.17 cm

Part II

The proportion of a fourteen year old's height to head length is 7 to 1. This relationship can be written as

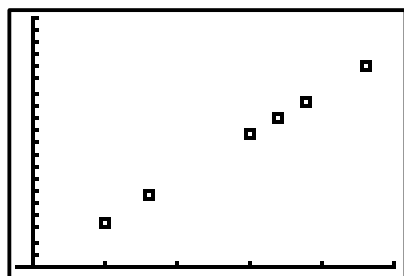
$$H = 7L$$

where H is the height and L is the length of the head.

1. Complete the following table using this model.

Head Length	Height
5	35
8	56
15	105
17	119
23	161
19	133

2. Using your graphing calculator as a guide, sketch the relationship displaying #1 above. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



```
WINDOW
Xmin=-1
Xmax=25
Xscl=5
Ymin=-1
Ymax=200
Yscl=10
Xres=1
```

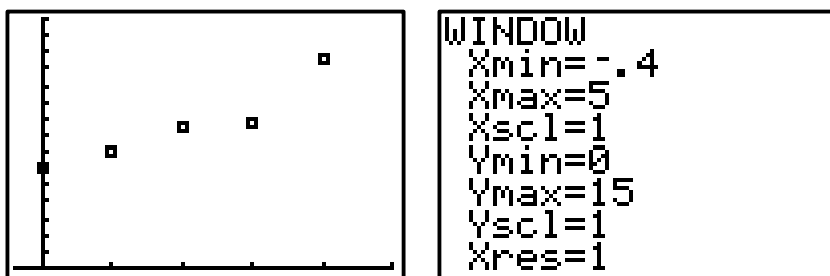
3. Describe the relationship between height and head length. Include units in your answer.
As head length increases in centimeters, so does the height of the fourteen-year-old.

Part III

You have been hired by ImagineLand County to analyze its population growth. Below is the population over the last five years.

Year	Population (in 100,000's)
1995	6
1996	7
1997	8.5
1998	8.8
1999	12.5

- Using your graphing calculator as a guide, sketch the relationship displaying the population. Let $x=0$ represent 1995. Make sure to label and scale the axes. Consider which variables are independent and which are dependent, and what parts of the coordinate plane make sense for this problem. Sketch your results below.



- Find the best-fit model (linear regression) of the data. Record your results.
Let x = number of years since 1995
Let y = population of ImagineLand County

 $y = 1.48x + 5.6$
- Identify the slope of your model. What does it mean in the context of this problem? Is the slope positive or negative, and what does that indicate in the context of this problem? Include units in your answer.
The positive slope indicates an increase of approximately 1.48 hundred thousand people per year in ImagineLand County.
- What will you report as the population growth rate to the ImagineLand County Council?
Rate of population growth = slope = 1.48 hundred thousand people per year
- Predict the population of ImagineLand County in the year 2005. Include mathematical analysis to support your findings.
2005 @ $x = 10$

$$y = 1.48(10) + 5.6$$
$$y = \underline{20.4 \text{ hundred thousand people}}$$

Rubric for Parts II and III

4	Student demonstrates a knowledgeable understanding of finding and using a linear function to model data.
	Student graph is labeled, scaled, and accurate.
	Student accurately uses model to find domain and range values.
	Student correctly identifies the dependent and independent variables.
	Student answers include units.
	Student correctly identifies and explains the rate of growth with units. (Part III).
3	Student demonstrates an understanding of finding and using a linear function to model data.
	Student graph is labeled or scaled, and accurate.
	Student uses model to find domain and range values, with minor mistakes.
	Student correctly identifies the dependent and independent variables.
	Most student answers include units.
	Student correctly identifies or explains the rate of growth with units. (Part III)
2	Student demonstrates some understanding of finding and using a linear function to model data.
	Student graph is labeled or scaled, and may have minor errors.
	Student uses model to find domain and range values, with mistakes.
	Student correctly identifies the dependent and independent variables.
	Some student answers include units.
	Student identifies slope. (Part III)
1	Student demonstrates little understanding of finding and using a linear function to model data.
	Student makes an attempt to graph.
	Student knows to use model to find domain and range values.
	Student attempts to identify the slope. (Part III)
0	Student makes no attempt to solve the problem.